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Relaxing Constraints from Lepton Flavor Violation in $5D$ Flavorful Theories

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Abstract

We propose new mechanisms for ameliorating the constraints on the Kaluza-Klein (KK) mass scale from charged lepton flavor violation in the framework of the Standard Model (SM) fields propagating in a warped extra dimension, especially in models accounting for neutrino data. These mechanisms utilize the extended five-dimensional ($5D$) electroweak gauge symmetry $\left[SU(2)_L \times SU(2)_R \times U(1)_X\right]$ which is already strongly motivated in order to satisfy electroweak precision tests in this framework. We show that new choices of representations for leptons under this symmetry (naturally) can allow small mixing angles for left-handed (LH) charged leptons and *simultaneously* large mixing angles for their $SU(2)_L$ partners, i.e., the LH neutrinos, with the neutrino data being accounted for by the latter mixings. Enhancement of charged lepton flavor violation by the large mixing angle observed in leptonic charged currents, which is present for the minimal choice of representations where the LH charged lepton and neutrino mixing angles are similar, can thus be avoided in these models. This idea might also be useful for suppressing the contributions to $B_{d,\,s}$ mixing in this framework and in order to suppress flavor violation from exchange of superpartners (instead of from KK modes) in $5D$ "flavorful supersymmetry" models. Additionally, the less minimal representations can provide custodial protection for shifts in couplings of fermions to Z and, in turn, further suppress charged lepton flavor-violation from tree-level Z exchange in the warped extra-dimensional scenario. As a result, $\sim O(3)$ TeV KK mass scale can be simultaneously consistent with charged lepton flavor violation *and* neutrino data, even without any particular structure in the $5D$ flavor parameters in the framework of a warped extra dimension.

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1 Introduction

The framework of a warped extra dimension was proposed in order to provide a solution to the Planck-weak hierarchy problem of the Standard Model (SM) [1]. With the SM gauge and fermions fields propagating in the extra dimension [2, 3, 4], it can also account for the flavor hierarchy of the SM via extra-dimensional profiles for SM fermions [3, 4]. Inherent to this approach is flavor violation from the resulting non-universal couplings of SM fermions to the Kaluza-Klein (KK) modes of the SM fields which are the new particles present in this framework [5]. In spite of an analog of Glashow-Iliopoulos-Maiani (GIM) mechanism being built-in to this framework [4, 6, 7], the lower limits on the mass scale of the KK gauge bosons from the flavor violation in the *quark* sector can still be $\sim O(5-10)$ TeV[2], depending on the details of the model [10, 11, 12] (see also [13, 14]). Whereas, flavor violation in the charged lepton sector requires a gauge KK mass scale at least as large as $\sim O(5)$ TeV *without consideration of neutrino data* [15]. It turns out that, in *minimal* models, enhancement of charged lepton flavor violation by the large mixing required in order to account for neutrino data results in constraints being even *stronger* than $\sim O(5)$ TeV [16].

Such a large gauge KK scale might imply a tension with a resolution of the Planck-weak hierarchy problem of the SM which requires a KK scale $\sim$ TeV. Also, signals from direct production of these KK modes at the Large Hadron Collider (LHC) (including upgrades) are then extremely challenging (if not unlikely). Therefore, it is very interesting to study mechanisms to ameliorate constraints from this flavor violation, thus allowing for a lower gauge KK mass scale. Recently, five-dimensional ($5D$) flavor symmetries (for both quark and lepton sectors) have been suggested for this purpose [17, 13, 18, 16, 19] such that a gauge KK scale as low as $\sim O(3)$ TeV might be allowed.

In this paper, we propose *alternative* mechanisms in order to suppress charged *lepton* flavor violation. The idea is to use new (less minimal) representations for leptons under the extended electroweak (EW) $5D$ gauge symmetry[3] – such an extended symmetry is *in any case* strongly motivated in order to suppress contributions to electroweak precision tests, in particular, the T parameter [21]. We demonstrate that

[2] A clarification about notation is in order here. The uncertainty in the bounds on KK mass scale from flavor violation denoted by the symbol "$\sim$ ..." comes from effects of modifications to the minimal model such as brane-localized kinetic terms for bulk fields [8] or replacement of the endpoint of the extra dimension by a "soft wall" [9]. Such variations are present even for limits on KK mass scale from electroweak precision tests. On the other hand, the symbol "$O(...)$" refers to uncertainties in the bounds from flavor violation due to presence of $O(1)$ factors in $5D$ Yukawa (which is an inherent feature of $5D$ flavor "anarchy") and due to the presence (typically) of more than one term (of similar size, but uncorelated) in the flavor-violating amplitude. Contributions to electroweak precision tests are not very sensitive to the latter types of effects and hence the "$O(...)$" factor is absent in that case.

[3] Very recently in reference [20], such non-minimal representations for leptons were studied in the context of gauge-Higgs unification, but their relevance for suppression of charged lepton flavor violation was not discussed.

- certain such choices of representations can allow small and large mixing angles to *naturally* co-exist for left-handed (LH) charged leptons and neutrinos, respectively, in spite of them being $SU(2)_L$ partners.

Thus it is possible (unlike in the minimal models) to avoid the large mixing angles required to explain the neutrino data from exacerbating charged lepton flavor violation.[4] As a result, even after including neutrino data, the constraint on the gauge KK scale can relax to the $\sim O(5)$ TeV value in the case without neutrino data. This new idea can suppress contributions to $B_{d,\,s}$ mixing (which, however, are not the dominant constraints from flavor violation in quark sector) as well by allowing LH down-type and up-type quark mixing angles to be parametrically different. Similarly it can also be applied to other extra-dimensional models (which account for flavor hierarchy via profiles for SM fermions in the extra dimension) in order to suppress flavor violation (especially in charged lepton sector).

Independently of the decoupling of large neutrino mixings from charged lepton sector, we show that

- such new representations can result in a custodial symmetry which protects shifts in coupling of SM fermions to Z in the framework of a warped extra dimension [24].

Hence flavor-violating Z couplings to leptons and the resulting tree-level Z exchange contributions to processes such as μ to e conversion in nuclei and $\mu \to 3\ e$ can be suppressed[5]. *Combining* the above two ideas, we show that it is possible to reduce the lower limit on gauge KK scale from charged lepton flavor violation (including neutrino data) down to $\sim O(3)$ TeV from $> O(5)$ TeV in the minimal model.

The outline of the rest of the paper is as follows. We begin with an overview of the framework of warped extra dimension (including both the quark and lepton sectors) and a *qualitative* outline of the problem of charged lepton flavor violation and the solutions proposed in this paper. Then we present *quantitative* estimates for charged lepton flavor violation in section 3, including how charged lepton flavor violation is enhanced by large neutrino mixing. The *central observations* of this paper are in the next two sections: in section 4, we show how to decouple mixings of the LH charged lepton and neutrino sectors, with many example representations for leptons under the extended $5D$ EW gauge symmetry: the general idea is illustrated in Fig. 2. In section 5 we consider a choice of profiles in order to obtain large neutrino mixing with mild tuning and then discuss the custodial protection mechanism which is critical to suppressing flavor-violating couplings to Z with

[4]Recently [22], it was shown that with a profile for the SM Higgs in the extra dimension (but still peaked near the endpoint of the extra dimension) [23] (instead of a δ-function localized Higgs) and with SM neutrinos being Dirac, it is possible to achieve a similar "decoupling" of LH neutrino and charged lepton mixing angles.

[5]The role of such a custodial symmetry in suppressing flavor violation in the *quark* sector was discussed in [11, 25].

this choice of profiles. A summary of various possibilities along these lines is provided in table 1. We conclude in section 6 with a brief discussion of signals for our new models. We also comment on the applicability of the mechanisms presented here to suppressing some contributions to flavor violation in the quark sector and to other extra-dimensional scenarios such as $5D$ "flavorful supersymmetry" (SUSY).

2 Overview

A slice of anti-de Sitter (AdS) space in $5D$ [1] provides a solution to the Planck-weak hierarchy problem of the SM. Basically, the warped geometry implies that the UV cut-off of the effective $4D$ theory depends on location in the extra dimension (y): $M_{4D\, eff.} \sim M_{5D} e^{-ky}$, where k is the AdS curvature scale, e^{-ky} is called the warp factor and M_{5D} is the fundamental $5D$ mass scale. The $4D$ graviton (zero-mode of the $5D$ gravitational field) is automatically localized near the $y = 0$ end of the extra dimension (hence called the Planck/UV brane). Suppose the Higgs sector is taken to be localized near the other end of the extra dimension (called the TeV/IR brane): $y = \pi R$, where R is the proper size (or radius) of the extra dimension – such a localization happens automatically if Higgs is the 5^{th} component of a $5D$ gauge field [26]. Then the Planck-weak hierarchy can be explained by a mild hierarchy between the AdS curvature radius ($\sim 1/k$ which is taken to be $\sim 1/M_{5D}$) and R: M_{Higgs} or $M_{weak} \sim M_{5D} e^{-k\pi R}$. Note that $M_{5D} \sim M_{Pl} \sim 10^{18}$ GeV is required in order to reproduce the observed ($4D$) Planck scale due to warp factor being 1 at the location of the $4D$ graviton. In turn, such a mild hierarchy between the proper size of the extra dimension and curvature radius: $k\pi R \sim \log(M_{Pl}/\text{TeV}) \sim 30$ can be stabilized by suitable mechanisms [27]. It is also that interesting that, based on the AdS/CFT correspondence [28], such a scenario is conjectured to be dual to SM Higgs being a composite of TeV-scale strong dynamics [29, 26].

2.1 SM in the bulk of warped extra dimension

Such a framework can also provide a solution to the flavor hierarchy of the SM if the SM fermions arise as zero-modes of fermions propagating in the extra dimension [3, 4]. Namely, the profiles of the SM fermions in the extra dimension are then controlled by their $5D$ masses. The crucial feature is that small variations in the $5D$ masses enable the SM fermions to have profiles which are peaked either near the Planck or TeV branes or are flat. This feature results in small/large/intermediate couplings of the SM fermions to the SM Higgs (which is localized near the TeV brane), simply based on overlaps of profiles in the extra dimension, i.e., without any hierarchy in the fundamental ($5D$) parameters (Yukawa couplings and $5D$ fermion masses).

SM gauge fields must also then originate as zero-modes of $5D$ fields ("SM in the bulk") [2, 4]

– it turns out that the SM gauge fields have a flat profile in the extra dimension. In addition to the zero-modes, the $5D$ fields have other, non-trivial excitations in the extra dimension (called Kaluza-Klein or KK modes) which appear as heavier particles from the $4D$ point of view. In the warped case, these KK modes turn out to be automatically localized near the TeV brane and have masses $\sim ke^{-k\pi R}$, i.e., at the $\sim$ TeV-scale. Thus all SM particles (except perhaps the SM Higgs) have KK modes in this scenario. So, contributions from these KK modes to precision tests of the SM can constrain this scenario. In particular, electroweak precision tests (EWPT) can be under control, using custodial symmetries to protect contributions to the T parameter [21] and the shift in $Zb\bar{b}$ coupling [24], even with KK masses of a few (or several) TeV [21, 30, 31].

2.2 Flavor violation

More relevant to this paper, there is flavor violation from exchange of KK modes which necessarily have non-universal couplings to the SM fermions (given that the flavor hierarchy is accounted for by SM fermions' non-universal profiles) [5]. However, there is an analog of the GIM mechanism of the SM which is automatic in this scenario since the non-universalities in the couplings of SM fermions to KK modes are of size of $4D$ Yukawa couplings (due to KK's having similar profile to Higgs) [4, 6, 7]. However, even in the presence of this RS-GIM mechanism, recently [10, 11] (see also [13, 14]) it was shown that the constraint on the KK mass scale from tree-level contributions of KK *gluon* to ϵ_K is quite stringent. In particular, for the model with the SM Higgs (strictly) localized on the TeV brane, the limit on the KK gluon mass scale from ϵ_K is $\sim O(10)$ TeV for the smallest allowed $5D$ QCD coupling obtained by *loop*-level matching to the $4D$ coupling with negligible tree-level brane kinetic terms. On the other hand, for larger brane kinetic terms such that the $5D$ QCD coupling (in units of the AdS curvature scale, k) is $\sim 4\pi$, the lower limit on KK gluon mass scale increases to $\sim O(40)$ TeV. In addition, the constraint on the KK gluon mass scale is weakened as the size of the $5D$ Yukawa (in units of k) is increased. However, this direction reduces the regime of validity of the $5D$ effective field theory (EFT): the above limits on KK gluon mass scale are for the size of $5D$ Yukawa such that about two KK modes are allowed in the $5D$ EFT.

Whereas, with a profile for the SM Higgs in the extra dimension (but which is still peaked near TeV brane [23], called a "bulk Higgs") and choosing the smallest allowed $5D$ QCD coupling and two KK modes in the $5D$ EFT, it was demonstrated in reference [12] that $\sim O(3)$ TeV KK gluon mass scale can be consistent with ϵ_K[6]. However, in the "two-site" model [32] (which is an economical approach to studying this framework by restricting to the SM fields and their first KK excitations),

[6] It was also argued in the same reference that a larger size of the $5D$ QCD coupling might in fact conflict with $5D$ perturbativity.

it was also shown in reference [12] that there is a “tension” between satisfying constraints form ϵ_K and BR($b \to s\gamma$) (the latter observable being sensitive to loop effects of KK modes). Thus, the limit on the mass scale for the new particles (assuming the heavy fermions and gauge bosons have same mass) must actually be a bit larger, namely, $\sim O(5)$ TeV to be consistent with this *combination* of constraints. Hence, it was also suggested reference [12] that the $5D$ models with a bulk Higgs can allow a similar, i.e., $\sim O(5)$ TeV, gauge KK scale to be consistent with the entire body of data on flavor violation in the quark sector.

Furthermore, $5D$ flavor symmetries in the quark sector can add more structure to the $5D$ model, for example, by relating the $5D$ (or bulk) fermion masses to the $5D$ Yukawa couplings and/or by enforcing degenerate bulk masses [17, 13, 19]. Such a reduction in the number of flavor parameters results in suppressed quark sector flavor violation. Also, by lowering the UV-IR hierarchy, i.e., $k\pi R$, it is possible to lower the gauge KK scale allowed by quark sector flavor violation [33], although in this paper we will always assume Planck-weak hierarchy, i.e., $k\pi R \sim 30$. Further studies of flavor violation, especially experimental signals, appear in references [34, 35].

In this paper, we focus instead on flavor violation and hierarchy of masses in the *lepton* sector. With*out* consideration of neutrino data, it was shown in reference [15] that the constraint from charged lepton flavor violation on gauge KK mass scale is $\sim O(5)$ TeV – such a strong constraint is mainly due to a tension between the two processes μ to e conversion in nuclei (which occurs at tree-level in this framework) and loop-induced $\mu \to e\gamma$. Note that this constraint was obtained for hierarchies in charged lepton masses being explained by the choice of hierarchical profiles near the TeV brane for both right-handed (RH) and LH charged leptons so that both RH and LH charged lepton mixing angles (given by ratio of respective profiles at the TeV brane) were set to be small (roughly *square root* of ratio of charged lepton masses).

2.3 Charged lepton flavor violation and neutrino data

However, including neutrino data, two new and distinct issues come up (see also related discussion in reference [16]):

(i) **Enhancement due to large mixing angle**: With the simplest representations under the extended bulk EW gauge symmetry, i.e., $SU(2)_L \times SU(2)_R \times U(1)_X$ (such an extension is typically required to satisfy EWPT) and a (strictly) TeV brane-localized Higgs, the charged lepton and neutrino (Dirac) masses originate from the *same* LH lepton bulk profiles evaluated at the TeV brane. Thus the mixing angles for LH charged leptons and neutrino are similar and, in turn, a combination these two mixing angles is what enters charged current lepton interactions. So, this mixing is required to be large in order to explain neutrino oscillation data.

Such large LH charged lepton mixing results in an enhancement of charged lepton flavor violation *relative to without considerations of neutrino data* as in [15] – as mentioned above, in reference [15] *both* RH and LH charged lepton mixing was set to be small. Thus, the gauge KK scale is constrained to be *larger* than $\sim O(5)$ TeV in order to be consistent with all the data, i.e., charged lepton flavor violation *and* neutrino mixings.

(ii) **Flat profiles for mild tuning**: For the case of a brane-localized Higgs, large LH neutrino mixing clearly requires non-hierarchical (i.e., with $\sim O(1)$ ratios) profiles for LH leptons near the TeV brane where the $4D$ Yukawa coupling originates. However, if the LH lepton profiles are peaked near the Planck brane, i.e., *exponentially* suppressed near the TeV brane, then it is clear that we need to tune the bulk masses (which control the exponentials) to be (almost) degenerate in order for the profiles near the TeV brane to be non-hierarchical.

If we require no tuning of bulk masses, then we might be forced to choose close-to-flat profiles for all generation LH leptons such that the profiles near the TeV brane can be non-hierarchical with only a *mild* tuning of bulk masses. However, such a choice results in a larger coupling of SM leptons to gauge KK modes (which are localized near IR brane) relative to the case of without considerations of neutrino data, i.e., where lepton profiles – both LH and RH – are peaked near the Planck brane (motivated by smallness of charged lepton masses). In turn, the larger couplings of leptons to KK modes enhance charged lepton flavor violation via tree-level Z exchange further, i.e., *in addition* to the effect of large LH charged lepton mixing angles mentioned in point (i) above.

Invoking $5D$ flavor symmetries is one way to solve the above problems [18, 16]. In particular, even if LH lepton profiles are peaked near the Planck brane, the (almost) degenerate LH lepton bulk masses required to give non-hierarchical profiles near TeV brane (and hence large mixing) are then enforced by a symmetry. Also, the resulting *universal* couplings of LH charged leptons to gauge KK modes (GIM mechanism) suppress LH charged lepton flavor violation from zero-KK gauge boson mixing: see top right-hand side of Fig. 1. Independently, such symmetries can relate bulk masses to $5D$ Yukawa couplings (just like for quarks discussed above) thus reducing the number of flavor parameters (i.e., adding structure). Hence LH charged lepton flavor violation from zero-KK *fermion* mixing (see top left-hand side of Fig. 1) and similarly $\mu \to e\gamma$ are suppressed as well.

Alternatively [22], for Higgs with a profile in the bulk (but still peaked near the TeV brane) [23] and with neutrinos being Dirac particles, neutrino masses of the observed size (i.e., required to account for neutrino oscillations) can arise from overlap near the *Planck* brane, whereas charged lepton masses originate (as usual) from the overlap of profiles near the TeV brane. Then the much smaller neutrino masses (relative to charged lepton) and large vs. small mixing in neutrino

and charged lepton sectors arise naturally. The point is that the LH lepton profiles can be non-hierarchical and large near the Planck brane (giving large mixing for LH neutrinos and ultra-small masses due to small Higgs profile at the Planck brane), while simultaneously being small and hierarchical near the TeV brane (giving small mixing and small masses for LH charged leptons).

2.4 New $5D$ gauge representations for leptons

In this paper, we propose an alternative to both the above ideas to suppress charged lepton flavor violation while obtaining large neutrino mixings. We still consider neutrino masses originating from near the TeV brane (say, Higgs is localized on the TeV brane or it leaks into the bulk, but not sufficiently for Dirac neutrino masses from overlap near the Planck brane to be of the observed size). The new idea is to use less minimal representations under the $SU(2)_R \times U(1)_X$ gauge symmetry. In particular, there are two new mechanisms as follows.

(a) **Decoupling large neutrino mixing from charged lepton masses**: the idea is that LH lepton zero-mode for each generation can arise as a *combination* of zero-modes from 2 different $5D$ multiplets: see Fig. 2. Such a scenario allows LH mixing angles to be parametrically different for charged leptons vs. neutrinos since the two mass matrices (and hence mixing angles) can originate from the two different components of the LH lepton zero-mode. In particular, mixing angles can then be small for charged vs. large for neutrinos. This novel possibility prevents large mixing angles in leptonic charged currents from infiltrating both tree-level Z exchange (giving μ to e conversion in nuclei) and loop-induced dipole operators (giving $\mu \to e\gamma$).

(b) **Custodial protection**: Independently, some choices of representations under the extended bulk EW gauge symmetry can result in a custodial symmetry for the shift in the couplings of leptons to Z (similar to one used to suppress shift in $Zb\bar{b}$ [24]). Such a symmetry can then suppress the flavor-violating couplings of leptons to Z and hence charged lepton flavor violation via tree-level Z exchange. Such a suppression is especially desirable if we choose (close-to-) flat profiles in order to generate large neutrino mixings without tuning (as mentioned in point (ii) in section 2.3). In such a case, the enhanced coupling of charged leptons to KK modes is still problematic for charged lepton flavor violation (as discussed in point (ii) in section 2.3), even if we obtain *small* charged lepton mixing angles using the idea in point (a) above.

It is in fact possible to *combine* the above two features for some choice of representations of leptons under the $5D$ EW gauge symmetry, resulting in $\sim O(3)$ TeV KK scale being consistent with charged

lepton flavor violation and large neutrino mixings (without any particular structure in the $5D$ flavor parameters). Various cases utilizing the above two ideas: (a) and/or (b) are listed in table 1.

3 Estimates for charged lepton flavor violation

In this section, we collect formulae for charged lepton flavor violation valid for the general case and then specialize to the models with neutrino masses. Since we are mainly concerned with parametric effects and mechanisms, estimates of these effects (i.e., formulae valid up to $O(1)$ factors) will suffice for our purpose. For more detailed formulae, the reader is referred to previous studies (see references [7, 15] for example). In addition to the effects of KK modes summarized below, there are also operators induced by physics at the cut-off of the $5D$ theory. For simplicity, we assume here that we have a bulk Higgs (but with a profile which is peaked near the TeV brane), where such cut-off effects can be shown to be smaller than KK-induced ones (see references [7, 15, 16]).

We first perform a KK decomposition for SM gauge and fermion fields setting the Higgs vev to zero. The $4D$ Yukawa coupling, i.e., the coupling of SM Higgs to two zero-mode fermions (say charged leptons), is given by:

$$Y_4\left(c_{e_L}, c_{e_R}\right) \sim Y_5\sqrt{k} f\left(c_{e_L}\right) f\left(c_{e_R}\right) \tag{1}$$

where Y_5 is $5D$ Yukawa coupling of mass dimension $-1/2$[7] and f's are ratio of zero-mode and KK profiles near the TeV brane:

$$f(c) \approx \begin{cases} \sqrt{\left(c-\frac{1}{2}\right)e^{k\pi R(1-2c)}} \text{ for } c > 1/2 \\ \sqrt{\frac{1}{2k\pi R}} \text{ for } c = 1/2 \\ \sqrt{\left(\frac{1}{2}-c\right)} \text{ for } c < 1/2 \end{cases} \tag{2}$$

where c is the $5D$ mass for the corresponding $5D$ fermion in units of k. We can show that the KK Yukawa coupling, i.e., the coupling of Higgs to two KK fermions, is given by:

$$Y_{KK} \sim Y_5\sqrt{k} \tag{3}$$

which (along with the above definition of f's) explains Eq. (1). Similarly, the coupling of Higgs to one zero-mode and one KK fermion is given by

$$Y_{mixed}(c) \sim Y_5\sqrt{k} f(c) \tag{4}$$

where c is that of the zero-mode fermion. Finally, the c-dependent part (which is the one relevant for flavor-violation) of the coupling of two zero-mode fermions to gauge KK mode is given by

$$g_4^{KK}(c) \sim g_{SM}\sqrt{k\pi R} f(c)^2, \tag{5}$$

[7] due to SM Higgs being in the bulk.

where we have used matching of $4D$ and $5D$ gauge couplings at the tree-level and without any brane-localized kinetic terms for gauge fields. We can use the above forumlae to estimate charged lepton flavor violation in this framework which is of two types: tree-level and loop processes which we now review in turn.

3.1 Tree-level

The tree-level flavor-violation occurs dominantly via Z exchange with the following flavor-violating Z couplings to leptons (we focus on μ and e in this paper, but the formulae can be easily generalized to the case of τ's):

$$\delta g^Z_{\mu_L e_L} \sim \left[\frac{M_Z^2}{M_{KK}^2} \times k\pi R + \frac{\left(Y_5\sqrt{k} \times v\right)^2}{M_{KK}^2}\right]\left[f\left(c_{\mu_L}\right)\right]^2 (U_L)_{12} \tag{6}$$

where 1st term originates from mixing between zero and KK gauge modes and 2nd term from fermion zero-KK mode mixing, both effects being induced by the Higgs vev: see Fig. 1.[8] Finally, $(U_L)_{12}$ denotes the mixing angle of the transformation from weak to mass basis for the charged leptons.

In particular, the assumption of a structureless or anarchic Y_5 (which we will make throughout this paper) implies that the mixing angles between charged leptons are given by ratio of profiles at the TeV brane (i.e., f's)

$$(U_L)_{ij} \sim \frac{f\left(c_{e_L\, i}\right)}{f\left(c_{e_L\, j}\right)} \text{ for } i < j \tag{7}$$

Similar formulae apply for $\delta g^Z_{\mu_R e_R}$ and RH charged lepton mixing.

In the special case of LH and RH charged lepton profiles being similar, i.e., hierarchies in charged lepton masses being explained *equally* by ratios of RH and LH profiles at the TeV brane, we find that both mixing angles are small and given by $(U_{L,\,R})_{12} \sim \sqrt{m_e/m_\mu}$ (based on Eqs. (1) and (7)). We then obtain

$$\delta g^Z_{\mu_L e_L},\ \delta g^Z_{\mu_R e_R} \sim \left[\left(\frac{M_Z^2}{M_{KK}^2} \times k\pi R\right)\frac{Y_{4\,\mu}}{Y_5\sqrt{k}} + \frac{Y_{4\,\mu} Y_5 \sqrt{k} v^2}{M_{KK}^2}\right]\sqrt{\frac{m_e}{m_\mu}} \tag{8}$$

Note that there are flavor-*preserving* shifts in couplings to Z which are given by similar formulae (except there are no mixing angles involved here).

[8] We assume small brane-localized kinetic terms for $5D$ fields so that the KK fermion and KK gauge masses are (almost) the same.

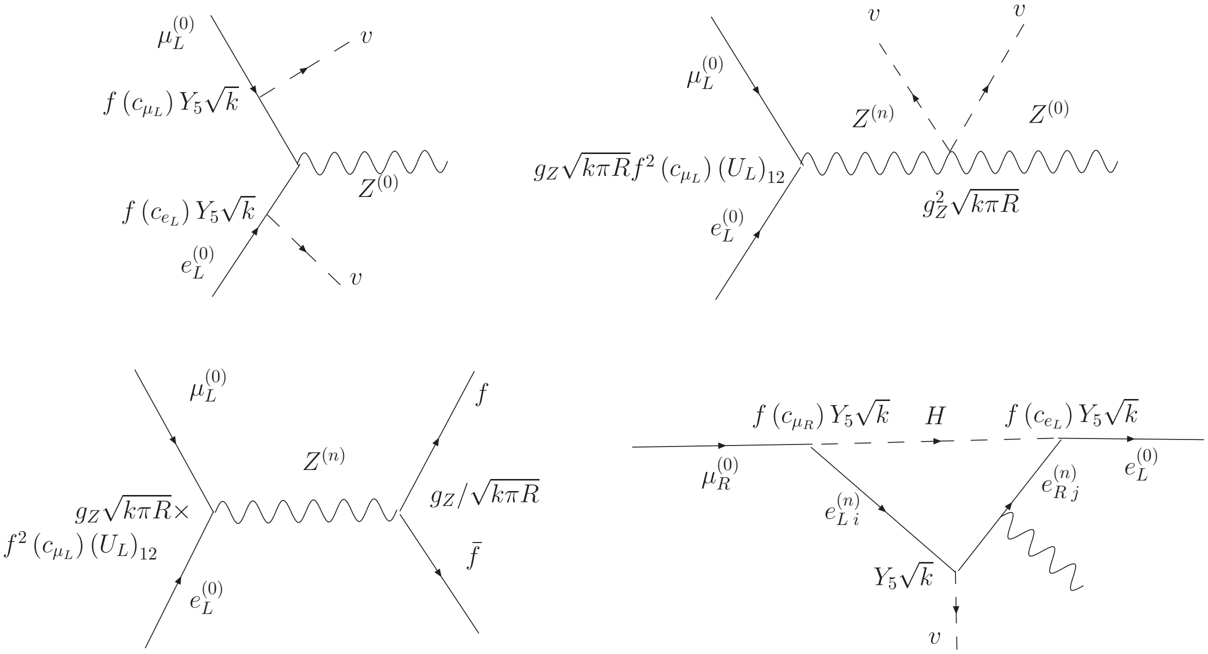

Figure 1: *Flavor violating couplings to Z generated by zero-KK mode fermion mixing (top left-hand side) and by zero-KK mode gauge mixing (top right-hand side), $\Delta F = 1$ 4-fermion operators generated by exchange of gauge KK modes (without mixing with the zero-mode, bottom left-hand side) and dipole operators generated by Higgs-KK fermion loops (bottom right-hand side).*

3.1.1 Direct KK Z exchange

The coefficient of the 4-fermion operator $\sim \overline{\mu_L}\gamma^\mu e_L \bar{f}\gamma_\mu f$ (where f = quark, lepton) generated by direct exchange of KK Z (i.e., without mixing with zero-mode Z) is given by (see Fig. 1)

$$\mathcal{A}^{\mathrm{KK}\,Z}\left(\mu_L \to e_L f\bar{f}\right) \;\;\sim\;\; \frac{g_Z^2}{M_{KK}^2}\left[f\left(c_{\mu_L}\right)\right]^2 \left(U_L\right)_{12} \tag{9}$$

where we have used the result that the flavor-*preserving* coupling of KK Z is $\sim g_{SM}/\sqrt{k\pi R}$ and similarly for $\mu_R \to e_R f\bar{f}$[9].

Comparing this effect to the one from Z exchange (based on Eq. 6), we see that the direct KK Z exchange is suppressed by $k\pi R \sim \log\left(M_{Pl}/\mathrm{TeV}\right)$. However, as mentioned earlier, we will invoke custodial symmetry to protect flavor violation from Z couplings, whereas direct KK Z exchange is not suppressed by this mechanism and thus might become relevant in these cases.

3.2 Loop

The coefficient of dipole operator: $e\, F_{\mu\nu}\overline{\mu_{L,\,R}}\sigma^{\mu\nu}e_{R,\,L}$ induced by loops of KK fermions and Higgs (including longitudinal W/Z), as in Fig. 1[10], is given by

$$\mathcal{A}\left(\mu_R \to e_L\gamma\right) \;\;\sim\;\; \frac{\left(Y_5\sqrt{k}\right)^2}{16\pi^2}\frac{m_\mu}{M_{KK}^2}\left(U_L\right)_{12} \tag{10}$$

and similarly for $\mu_L \to e_R\gamma$.

Again, in the case of LH and RH being similar, we find

$$\mathcal{A}\left(\mu_R \to e_L\gamma\right),\; \mathcal{A}\left(\mu_L \to e_R\gamma\right) \;\;\sim\;\; \frac{\left(Y_5\sqrt{k}\right)^2}{16\pi^2}\frac{m_\mu}{M_{KK}^2}\sqrt{\frac{m_e}{m_\mu}} \tag{11}$$

Note that there is some tension between tree-level and loop processes from the size of Y_5 in the sense that the former (1st term in Eq. (8)) is enhanced for small Y_5 while the latter (Eq. (11)) is suppressed in this limit. Without considerations of neutrino data (in particular, not taking into account the large charged current mixing which is a combination of LH charged lepton and neutrino mixing angles), we can assume LH and RH charged lepton profiles are similar, i.e., both sets of profiles are hierarchical at the TeV brane and mixing angles are small as in Eqs. (8) and (11). This is the case studied in reference [15] with the result that the least constrained scenario (i.e., lowest

[9] KK photon will also induce similar effects. And, in the models with extended EW gauge symmetry, there is an addition neutral gauge boson tower (denoted by Z'), i.e., the combination of the $U(1)$ subgroup of $SU(2)_R$ and $U(1)_X$ which is orthogonal to the hypercharge gauge symmetry, $U(1)_Y$. However, flavor-*preserving* couplings of Z' to light SM fermions which are localized near the Planck brane are suppressed compared to the coupling to KK Z – roughly the former couplings are of size given by $4D$ Yukawa couplings.

[10] It turns out that the loops with KK W/Z or transverse SM W/Z and KK fermions are approximately aligned with $4D$ Yukawa and hence do not contribute to $\mu \to e\gamma$ [7, 15].

KK scale) is with $Y_5\sqrt{k} \sim O(1)$ which still requires $\sim O(5)$[11] TeV gauge KK mass scale in order to be consistent with charged lepton flavor violation data. It turns out that the flavor-*preserving* shifts in Z couplings to leptons are then quite safe.

3.3 Enhanced effects due to fitting neutrino data

Having estimated that $\sim O(5)$ TeV gauge KK mass scale can be consistent with charged lepton flavor violation with*out* considering neutrino masses, we next discuss how incorporating neutrino data affects these estimates. It is usually assumed that LH profile (at the TeV brane) governing charged lepton mass is the same as that for neutrino mass (for each generation) because LH lepton zero-mode originates from a single $5D$ multiplet, i.e., $f\left(c_{e_L\,i}\right) = f\left(c_{\nu_L\,i}\right) \equiv f\left(c_{L\,i}\right)$. Clearly, along with the assumption of a anarchic Y_5, the mixing angles (appearing in the bi-unitary transformation to go from weak to mass basis) for LH charged leptons and neutrinos are then of the same order (but not exactly the same) in these minimal models. The reason for this feature is that the mixing angles are dictated by the ratios of profiles of the three L zero-modes near the TeV brane: see Eq. (7). In turn, the neutrino oscillation data (i.e., large mixing in leptonic charged currents which is a combination of LH charged lepton and neutrino mixing) then requires this LH lepton mixing angle to be large.

Thus we make the following change compared to the case without neutrino masses considered in reference [15]: $(U_L)_{12} \sim \sqrt{m_e/m_\mu} \rightarrow\sim O(1)$, which must result from no hierarchies in LH lepton profiles near the TeV brane, i.e., $f\left(c_{L\,1}\right) \sim f\left(c_{L\,2}\right) \sim f\left(c_{L\,3}\right)$.[12] Thus, once we include neutrino data, it seems that LH and RH profiles cannot be chosen to be similar for charged leptons. In turn, no hierarchies in LH charged lepton profiles at the TeV brane implies that the hierarchies in charged lepton masses are then explained *entirely* by hierarchies in RH charged lepton profiles at the TeV brane, resulting in RH charged lepton mixing actually being smaller than in the case assumed in reference [15]: $(U_R)_{12} \sim \sqrt{m_e/m_\mu} \rightarrow m_e/m_\mu$ (based on Eqs. (1) and (7)).

In short, with the above changes for mixing angles in the estimates for charged lepton violation from sections 3.1 and 3.2, we find that the "best" case, i.e., with lowest KK mass scale, allowed by charged lepton flavor violation and taking into account the constraints from flavor-*preserving* shifts $\delta g^Z_{e_{R\,i}e_{R\,i}}$, $\delta g^Z_{e_{L\,i}e_{L\,i}}$, $\delta g^Z_{\nu_{L\,i}\nu_{L\,i}} \stackrel{<}{\sim}$ a few 0.1% is the following:

- $M_{KK} \sim O(10)$ TeV for $Y\sqrt{k} \sim O(0.6)$, $f\left(c_{\tau_R}\right) \sim O(1)$ and $f\left(c_{L\,i}\right) \sim O(0.015)$

[11]Note that this is the limit on KK scale obtained by considering only *one* term at a time in the flavor-violating amplitude (from among several uncorrelated terms of similar size), whereas some of the limits quoted in reference [15] were based on the *combined* effect of all terms in this amplitude (for a certain choice of relative phases between the various terms).

[12]We assume these f's are *not* exactly equal since that would require a tuning of c's.

so that $c_{L\,i} > 1/2$. Thus, the LH lepton profiles are peaked near the Planck brane so that we do need to choose $c_{L\,i}$'s to be close to each other in order to achieve the (exponentially suppressed) profiles near the TeV brane being non-hierarchical: see Eq. (2) (we will return to this issue later).

In more detail (this discussion is an elaboration of point (i) of section 2.3 and will be useful later), there are more than one "count" of enhancement of charged lepton flavor violation once we include neutrino masses relative to the case without neutrino masses:

I For $\mu_R \to e_L \gamma$, we have enhancement from $(U_L)_{12} \sim \sqrt{m_e/m_\mu} \to\sim O(1)$, although $\mu_L \to e_R \gamma$ is suppressed compared to the case without neutrino masses due to $(U_R)_{12} \sim \sqrt{m_e/m_\mu} \to\sim m_e/m_\mu$. There is a similar enhancement and suppression for the two tree-level contributions, i.e., $\delta g^Z_{\mu_L e_L}$ and $\delta g^Z_{\mu_R e_R}$, respectively.

II (A) Another count of enhancement (relative to case without neutrino masses) for $\delta g^Z_{\mu_L e_L}$ comes due to three $f\left(c_{L\,i}\right)$'s being similar, i.e., $f\left(c_{L\,2}\right)$ in Eq. (6) is clearly dictated by m_τ (instead of depending only on m_μ earlier) since it is now (roughly) similar to $f\left(c_{L\,3}\right)$ and hence can be larger.

II (B) Moreover, we might try to choose smaller Y_5 in order to to suppress $\mu_R \to e_L \gamma$ (see Eq. (10)), keeping several TeV KK mass scale (in the light of point I above). Such a smaller Y_5 implies that, in order to obtain correct m_τ, $f\left(c_{L\,3}\right)$ (and hence $f\left(c_{L\,2}\right)$ also), in turn, might have to be larger than in the case without neutrino masses in reference [15].

Of course, we are free to choose RH and LH charged lepton profiles to be different (unlike the case considered in reference [15]), in particular, we can increase $f\left(c_{\tau_R}\right)$ in order to make $f\left(c_{L\,3}\right)$ smaller while keeping m_τ fixed. Hence $\delta g^Z_{\mu_L e_L}$ can be smaller, avoiding the enhancements in point II (A) and (B) above. However, a too large $f\left(c_{\tau_R}\right)$ is constrained by $\delta g^Z_{\tau_R \tau_R} \lesssim$ a few 0.1 %. The best case is then obtained by choosing $f\left(c_{\tau_R}\right)$ and Y_5[13] such that constraint on M_{KK} is the same from three observables: $\delta g^Z_{\tau_R \tau_R}$, $\mathcal{A}\left(\mu_R \to e_L \gamma\right)$ and $\delta^Z_{\mu_L e_L}$ (it can be checked that the other processes – both flavor-violating and flavor-preserving – are more easily satisfied and so are not the bottlenecks). It is such an analysis which shows that the lowest allowed KK scale is $O(10)$ TeV (as mentioned above).

4 Decoupling LH neutrino and charged lepton mixing: new "selection rules" for Yukawa couplings

Clearly, the "cornering" involving the various observables discussed above can be simply avoided if the LH profiles (at the TeV brane) which govern the charged lepton and neutrino masses, and

[13] $f\left(c_{L\,3}\right)$ – and hence $f\left(c_{L\,1,\,2}\right)$ (up to $\sim O(1)$ factor) – is then fixed by m_τ.

hence the corresponding mixing angles, could actually be different. A priori, it might seem difficult to achieve this scenario (due to LH charged lepton and neutrino being $SU(2)_L$ partners) but remarkably it is possible as follows! The central idea is that the SM $SU(2)_L$ doublet LH lepton ($l^{(0)}$) (for one generation) is actually a *combination* of zero-mode $SU(2)_L$ doublets from two $5D$ multiplets with different profiles such that the charged lepton masses originate from one component of this zero-mode, whereas obtaining neutrino masses requires using the other component. Although we focus on leptons here, a similar argument can apply to quarks in order to obtain parametrically different mixing angles for LH down-type vs. up-type quarks.[14]

4.1 General Case

Let us see how to implement this idea in detail. We will begin with a discussion of the general case which will enable us to see how to apply it to other extra-dimensional models and also to the quark sector. There are **three** main ingredients of this idea (which is summarized in Fig. 2):

(1) Suppose the $5D$ gauge symmetry is *extended* beyond the SM gauge symmetry and is reduced to the SM gauge symmetry by boundary conditions at the Planck brane (or equivalently by a large scalar vev on the Planck brane). In other words, the gauge symmetry of the $4D$ effective theory (at the level of zero-modes) is only the SM symmetry, but it is a *subgroup* of the $5D$ gauge symmetry.

(2) Consider two $5D$ fermion multiplets, L_e and L_ν, which transform *differently* under the $5D$ gauge symmetry (and hence cannot mix in the bulk/on the TeV brane). Moreover, these two multiplets contain zero-modes (to begin with: see later) – denoted by $l^{(0)}_{e,\,\nu}$, respectively – which transform like LH leptons (i.e., identically) under the SM EW gauge symmetry. Hence these two zero-modes can mix on the Planck brane (*only*) since the Planck brane respects only the SM (and not the full $5D$) gauge symmetry.

Specifically, one combination of the two zero-modes is given a (Planck-scale) mass with a fermion localized on the Planck brane, $l'_{R\,i}$ (effectively this combination of the two $5D$ multiplets has Dirichlet boundary condition on the Planck brane):

$$\mathcal{L}_{\text{UV brane}} \ni \overline{l'_{R\,i}} \left(\sin\alpha_i l^{(0)}_{e\,i} - \cos\alpha_i l^{(0)}_{\nu\,i} \right) \tag{12}$$

The orthogonal combination of the two zero-modes is left over as the only massless mode and is then identified with the SM LH lepton:

$$l^{(0)}_i = \cos\alpha_i l^{(0)}_{e\,i} + \sin\alpha_i l^{(0)}_{\nu\,i} \tag{13}$$

[14] In fact, two different $5D$ $SU(2)_L$ multiplets have already been used in references [31, 10, 19] in order to obtain up and down-type quark masses, but the implication for decoupling the down-type quark mixing angles from the up-type was not specifically considered in these references.

For simplicity, we neglect flavor mixing in Eqs. (12) and (13).[15] The gauge couplings of the SM fermion to leading order (i.e., couplings to the gauge zero-mode) are obviously not affected by such a combination of the fermion zero-modes.

(3) Moreover, the representations of the RH charged leptons and neutrinos under the $5D$ gauge symmetry are *chosen* to be such that their couplings to Higgs – localized near the TeV brane – must involve the two different components of the $l^{(0)}$. The reason for these new "selection rules" is that the Higgs couplings (in general, all bulk and TeV brane interactions) respect the full $5D$ gauge symmetry (which is, again, larger than the SM one). Therefore, charged lepton and neutrino masses depend on the different profiles of the two components of $l^{(0)}$, giving different LH mixing angles.

Note that it is the enlarged $5D$ *symmetry* which forces this "decoupling" of LH profiles involved in the charged lepton masses from those involved in the neutrino masses – obviously the SM/$4D$ symmetry would allow charged lepton and neutrino masses to proceed via the *same* component of the $l^{(0)}$. Also, the Higgs couplings must satisfy the larger $5D$ gauge symmetry even in the *minimal* case where we require (for simplicity) that the SM LH lepton originates as zero-mode of a *single* $5D$ field – it is just that in this case these selection rules then get translated into specific representations for the RH charged leptons and neutrinos under the $5D$ gauge symmetry.

4.2 Examples with extended electroweak symmetry: $SU(2)_L \times SU(2)_R \times U(1)_X$

Specifically, consider $SU(2)_L \times SU(2)_R \times U(1)_X$ as the $5D$ EW gauge symmetry with $U(1)_Y$, being a combination of $U(1)_X$ and $U(1)$ subgroup of $SU(2)_R$, i.e., $Y = T_{3R} + X$. As already mentioned, such an extension is motivated by satisfying EWPT, in particular, the constraint from the T parameter. (However, this idea can be generalized to other extended $5D$ gauge symmetries, such as a grand-unified one.) The SM Higgs transforms as $(\mathbf{2}, \mathbf{2})_0$, where 1st/2nd symbol in (...) is the representation under $SU(2)_{L,R}$ symmetry and the subscript denotes the charge under $U(1)_X$.

The two different $5D$ multiplets which will constitute the $l^{(0)}$ transform as $L_e : (\mathbf{2}, \mathbf{r_{L_e}})_{X_E}$ and $L_\nu : (\mathbf{2}, \mathbf{r_{L_\nu}})_{X_N}$, respectively. Note that, in general, $\mathbf{r_{L_{e,\,\nu}}} \neq \mathbf{1}$ so that each multiplet can contain *more than one* $SU(2)_L$ doublet. We must choose the various charges such that $Y = T_{3R} + X = -1/2$ (i.e., the Y for the SM LH lepton) for one $SU(2)_L$ doublet contained in each of the two multiplets,

[15] In any case, we can show that such effects not significant. For the mixing of $l_\nu^{(0)}$ components of different generations on the Planck brane, this conclusion is due to either to custodial protection for the resulting flavor-violating couplings to Z or to the choice $f\left(c_{L_\nu}\right) \ll 1$ (see discussion in section 5). Therefore, we are left with the (mass) mixing of $l_e^{(0)}$ components which can be shown to be equivalent to mixing via Planck brane localized *kinetic* terms. Such mixing appears even in the minimal models (and even without consideration of neutrino masses) where LH lepton arises from a single $5D$ field. Flavor violation due to such kinetic terms (even if they are $\sim O(1)$, i.e., comparable to bulk contributions) can also be shown to be small.

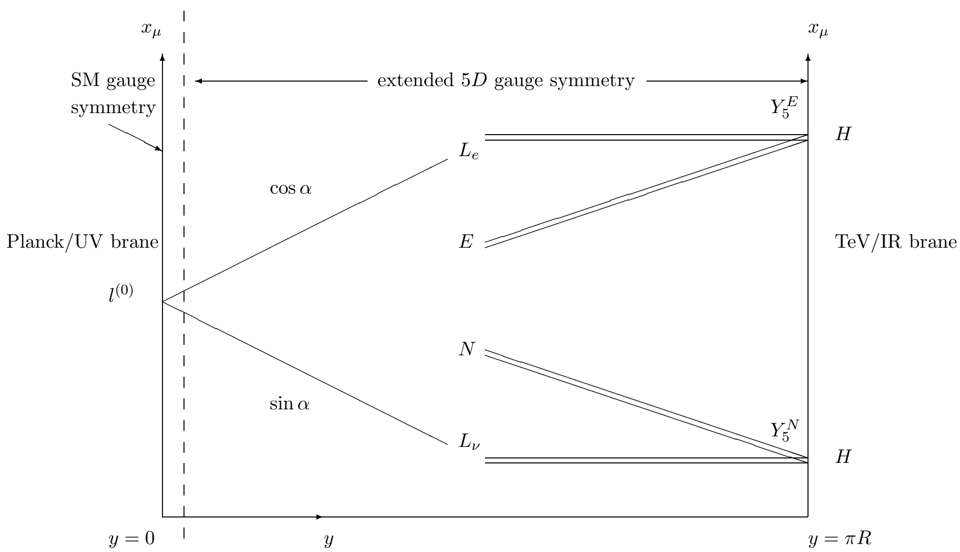

Figure 2: *The mechanism for decoupling LH charged lepton and neutrino mixing angles. The single lines denote mixing of the zero-modes from the bulk $SU(2)_L$ doublet multiplets, $L_{e,\,\nu}$ on the Planck brane (which respects only the $4D$/SM gauge symmetry) and double lines denote their couplings (which respect the enlarged $5D$ gauge symmetry) to Higgs, H and the bulk $SU(2)_L$ singlet multiplets, E and N.*

L_e and L_ν. Moreover, we choose only these two components of the two multiplets to have zero-modes, i.e., to begin with, we choose Neumann boundary condition on both branes only for the corresponding $5D$ fields. Thus these two zero-modes correspond to the $l^{(0)}_{e,\,\nu}$ in the general discussion above. Any extra "would-be" $SU(2)_L$ doublet zero-modes from the rest of the multiplets can be projected out using Dirichlet boundary condition for the corresponding $5D$ fields on the Planck brane. Next, these two zero-modes mix as in Eqs. (12) and (13) since on the Planck brane only $U(1)_Y$ is preserved so that finally we are left with only one $SU(2)_L$ massless doublet (per generation). Equivalently, ultimately only the combination of the two $5D$ multiplets L_e and L_ν in Eq. (13) has Neumann boundary condition on the Planck brane.

The SM e_R can arise as zero-mode of a *single* $5D$ multiplet (denoted by E) and ν_R as zero-mode of a different (single) $5D$ multiplet (denoted by N).[16] As discussed above, we must choose representations for the RH charged lepton and neutrino $5D$ multiplets under the bulk EW gauge symmetry, $(\mathbf{1}, \mathbf{r_E})_{X_E}$ and $(\mathbf{1}, \mathbf{r_N})_{X_N}$[17] (respectively), such that charged lepton and neutrino masses must proceed via the $l^{(0)}_e$ and $l^{(0)}_\nu$ components of $l^{(0)}$. Schematically, we desire

$$\text{general case}: \; Y_E: \; \overline{(\mathbf{2}, \mathbf{r_{L_e}})_{X_E}}\, (\mathbf{1}, \mathbf{r_E})_{X_E}\, (\mathbf{2}, \mathbf{2})_\mathbf{0} \quad Y_N: \; \overline{(\mathbf{2}, \mathbf{r_{L_\nu}})_{X_N}}\, (\mathbf{1}, \mathbf{r_N})_{X_N}\, (\mathbf{2}, \mathbf{2})_\mathbf{0} \tag{14}$$

Thus, we require

$$\mathbf{r_{L_e}} \times \mathbf{r_E} \ni \mathbf{2} \quad \mathbf{r_{L_\nu}} \times \mathbf{r_N} \ni \mathbf{2} \tag{15}$$

so that E can couple to L_e and Higgs (and similarly for N), but

$$X_E \neq X_N \; or \quad \mathbf{r_{L_e}} \times \mathbf{r_N} \not\ni \mathbf{2}, \; \mathbf{r_{L_\nu}} \times \mathbf{r_E} \not\ni \mathbf{2} \tag{16}$$

so that E cannot couple to L_ν and Higgs (and similarly for N). Note that these Higgs couplings must preserve the enlarged, i.e., $SU(2)_L \times SU(2)_R \times U(1)_X$, bulk gauge symmetry, and not just the SM symmetry. Hence, if Eq. (16) is satisfied, then charged lepton mass cannot proceed via the $l^{(0)}_\nu$ component of $l^{(0)}$ and vice versa. Clearly, then the hierarchies in charged lepton and neutrino masses are set by the hierarchies in profiles (at the TeV brane) of ($l^{(0)}_e$, $e^{(0)}_R$) and ($l^{(0)}_\nu$, $\nu^{(0)}_R$), respectively. In particular, large (small) LH mixing desired for charged leptons (neutrinos) is achieved via small (large) hierarchies in the profiles at the TeV brane of the $l^{(0)}_e$ ($l^{(0)}_\nu$) components of the SM LH lepton.

[16] Clearly we cannot do such a splitting for e_L and ν_L due to $SU(2)_L$ symmetry being preserved at the zero-mode/$4D$ level (of course before Higgs acquires a vev).

[17] As for the case of $L_{e,\,\nu}$, in general, we have $r_{E,\,N} \neq \mathbf{1}$ so that there are extra components (other than the SM e_R and ν_R) in the $5D$ E and N multiplets. In fact, it is possible that the $5D$ multiplet E has a component with quantum numbers of ν_R and vice versa. "Unwanted" zero-modes for such components will have to be projected out using Dirichlet boundary condition on the Planck brane. Again, only SM gauge symmetry is preserved on the Planck brane so that such "splitting" of multiplets can be realized.

4.2.1 $X = 1/2\ (B-L)$

We will now discuss explicit examples. Begin with the minimal case: $X = 1/2\ (B-L)$ and only one $5D$ LH lepton multiplet, $L:\ (\mathbf{2},\mathbf{1})_{\mathbf{-1/2}}$ (of course this case will have the constraints discussed in section 3.3). The RH charged leptons and neutrinos can be obtained from different $(\mathbf{1},\mathbf{2})_{\mathbf{-1/2}}$ $5D$ multiplets (labeled with superscripts e and ν below) with the extra states in each multiplet having no zero-modes due to Dirichlet boundary condition on the Planck brane. Charged lepton and Dirac neutrino masses then both arise from

$$\text{Case (0)}:\quad Y_E \text{ and } Y_N:\ \overline{(\mathbf{2},\mathbf{1})_{\mathbf{-1/2}}}(\mathbf{1},\mathbf{2})^{\mathbf{e},\,\nu}_{\mathbf{-1/2}}(\mathbf{2},\mathbf{2})_{\mathbf{0}} \tag{17}$$

giving similar (and hence large) mixing angle for LH charged leptons and neutrinos and resulting in the KK mass limit $\sim O(10)$ TeV. Recall that this choice of parameters additionally requires tuning of $c_{L\,i}$'s in order to obtain large mixings (as mentioned earlier).

Note, however, that, even with the choice $X = 1/2\ (B-L)$, all that we require is $T_{3R} = 0, -1/2, +1/2$ for L, E and N, respectively, i.e., we are not forced to choose L to be singlet of $SU(2)_R$ or E, N to be doublets of $SU(2)_R$ – it suffices to choose integer and half-integer spin representations of $SU(2)_R$, respectively, for them. Thus we could instead choose $L_e:(\mathbf{2},\mathbf{1})_{\mathbf{-1/2}}$, $L_\nu:(\mathbf{2},\mathbf{5})_{\mathbf{-1/2}}$, $E:(\mathbf{1},\mathbf{2})_{\mathbf{-1/2}}$ and $N:(\mathbf{1},\mathbf{4})_{\mathbf{-1/2}}$ in order to satisfy the 2nd condition in Eq. (16) for different mixing angles (even though $X_E = X_N$ in this case). So, we have

$$\text{Case (1)}:\quad Y_E:\ \overline{(\mathbf{2},\mathbf{1})_{\mathbf{-1/2}}}(\mathbf{1},\mathbf{2})_{\mathbf{-1/2}}(\mathbf{2},\mathbf{2})_{\mathbf{0}}\quad Y_N:\overline{(\mathbf{2},\mathbf{5})_{\mathbf{-1/2}}}(\mathbf{1},\mathbf{4})_{\mathbf{-1/2}}(\mathbf{2},\mathbf{2})_{\mathbf{0}} \tag{18}$$

4.2.2 $X \neq 1/2\ (B-L)$

In general, $X \neq 1/2\ (B-L)$ such that even $X_E \neq X_N$ is possible. For example, $L_e:(\mathbf{2},\mathbf{1})_{\mathbf{-1/2}}$, $L_\nu:(\mathbf{2},\mathbf{2})_{\mathbf{0}}$, $E:(\mathbf{1},\mathbf{2})_{\mathbf{-1/2}}$ and $N:(\mathbf{1},\mathbf{3})_{\mathbf{0}}$ so that

$$\text{Case (2)}:\quad Y_E:\ \overline{(\mathbf{2},\mathbf{1})_{\mathbf{-1/2}}}(\mathbf{1},\mathbf{2})_{\mathbf{-1/2}}(\mathbf{2},\mathbf{2})_{\mathbf{0}}\quad Y_N:\overline{(\mathbf{2},\mathbf{2})_{\mathbf{0}}}(\mathbf{1},\mathbf{3})_{\mathbf{0}}(\mathbf{2},\mathbf{2})_{\mathbf{0}} \tag{19}$$

As a final example of decoupling of LH charged and neutrino mixing angles, we choose $L_e:(\mathbf{2},\mathbf{2})_{\mathbf{-1}}$, $L_\nu:(\mathbf{2},\mathbf{2})_{\mathbf{0}}$, $E:(\mathbf{1},\mathbf{1})_{\mathbf{-1}}$ and $N:(\mathbf{1},\mathbf{1})_{\mathbf{0}}$ so that

$$\text{Case (3)}:\quad Y_E:\ \overline{(\mathbf{2},\mathbf{2})_{\mathbf{-1}}}(\mathbf{1},\mathbf{1})_{\mathbf{-1}}(\mathbf{2},\mathbf{2})_{\mathbf{0}}\quad Y_N:\overline{(\mathbf{2},\mathbf{2})_{\mathbf{0}}}(\mathbf{1},\mathbf{1})_{\mathbf{0}}(\mathbf{2},\mathbf{2})_{\mathbf{0}} \tag{20}$$

The motivation for the above two cases will be discussed later.

5 Large neutrino mixing with only mild tuning

The above idea of decoupling large neutrino mixing from LH charged lepton sector resolves only part of the problem discussed in section 3.3, i.e., count (I) only, namely, the enhancing effect of

mixing angles. In particular, the EW structure of the charged lepton dipole operator is similar to the masses so that the $l_\nu^{(0)}$ component of $l^{(0)}$ does not enter the amplitude for $\mu \to e\gamma$. Hence, the estimates for $\mu \to e\gamma$ are similar to the case without neutrino masses.

However, the flavor violation via tree-level Z exchange is still modified (relative to the case without neutrino masses) as follows. The point is that, although the $l_\nu^{(0)}$ component of $l^{(0)}$ does not determine the LH charged lepton mixing *angles*, it does contribute to the coupling of LH charged leptons to KK Z which is (approximately) diagonal in generation space in the weak basis for leptons[18]. And, even though we have $f\left(c_{L_\nu\,1}\right) \sim f\left(c_{L_\nu\,2}\right) \sim f\left(c_{L_\nu\,3}\right)$, we are not assuming *strict* universality of these f's (which would require tuning or a symmetry) so that couplings of SM LH leptons to KK Z (see Eq. (5)) induced by their $l_\nu^{(0)}$ components do differ by $\sim O(1)$ factors. In turn, via KK Z-zero-mode Z mixing (see top right-hand side diagram in Fig. 1), these couplings to KK Z result in *non*-universal shifts (relative to the $Z^{(0)}$ coupling) in the coupling of LH charged leptons to SM Z. In the weak basis, these shifts in the couplings to Z are still diagonal in generation space. Recall that these non-universal shifts in coupling to Z then get converted into flavor-*violating* coupling to Z up on going from weak to mass basis, as in 1st term of Eq. (6) (albeit with small mixing angle in this case). Therefore, depending on the size of $f\left(c_{L_\nu}\right)$, the $l_\nu^{(0)}$ component can still be important for tree-level charged lepton flavor violation via Z exchange.

We can then distinguish two cases (this discussion is an elaboration of point (ii) of section 2.3). If $c_{L_\nu} > 1/2$, then we have $f\left(c_{L_\nu}\right) \ll 1$, i.e., the $l_\nu^{(0)}$ are peaked near the Planck brane. In particular, we can choose $f\left(c_{L_\nu}\right) \stackrel{<}{\sim} f\left(c_{L_e\,2}\right)$, where the latter parameter determines m_μ, so that the effect of $l_\nu^{(0)}$ component in tree-level charged lepton flavor violation (in zero-KK gauge mode mixing) is smaller than that of the L_e component[19]. Then, the tree-level charged lepton flavor violation is also same as in the case without neutrino masses, i.e., generically KK mass limit is $O(5)$ TeV. However, obtaining non-hierarchical profiles near TeV brane for the $l_\nu^{(0)}$ component in order to generate large LH neutrino mixings then requires (almost) degenerate bulk masses, i.e., tuning, due to the profiles' exponential sensitivity to the bulk masses: see Eq. (2). Specifically, we need a splitting in c of $\sim 1/\left(k\pi R\right) \sim 0.03$ if we require (at most) a factor of ~ 3 hierarchy in the profiles at the TeV brane for $c > 1/2$.

So, we consider instead $c_{L_\nu} \stackrel{<}{\sim} 1/2$ such that f's have a milder (power-law instead of exponential) dependence on c_L (see Eq. (2)), i.e., the $l_\nu^{(0)}$ have a flat/peaked near TeV brane profile. Thus there is no need for any tuning of bulk masses in this case in order to obtain large LH neutrino mixing.

[18]Here, we are assuming that the off-diagonal couplings of leptons (in this basis) to *KK Z* which are induced via brane-localized kinetic terms are small. Similarly, such off-diagonal effects generated by zero-KK fermion mixing (see top left-hand side of Fig. 1, with $Z^{(0)}$ replaced by $Z^{(n)}$) are also suppressed, assuming $Y_5\sqrt{k} \sim O(1)$.

[19]Similarly, the $l_\nu^{(0)}$ component of LH lepton also contributes to off-diagonal couplings to SM Z (already in the weak basis for leptons) via zero-KK fermion mode mixing. This effect can also be suppressed by the choice of $c_{L_\nu} > 1/2$.

However, the case $c_{L_\nu} < 1/2$ is strongly constrained by EWPT (independent of flavor-violation) even for several TeV KK scale. Specifically, due to the enhanced coupling of LH leptons to KK Z (see Eq. (5)), the flavor-*preserving* shift of the couplings of leptons to Z[20] and 4-fermion operators induced by *direct* KK gauge exchange become too large[21].

As a compromise, we are then led to considering $c_{L_\nu} \sim 1/2$ (i.e., close-to-flat profiles), but still not degenerate: for example, based on Eq. (2), we find that

- $c = 0.525 \leftrightarrow 0.45$ gives only a factor of ~ 3 (i.e., not larger) hierarchy in the profile at the TeV brane (see Eq. (2)) which will still give large LH neutrino mixing.

The splitting in bulk masses, $\Delta c \sim 0.075$, is still small, but it is similar to the splitting of bulk masses in the quark sector (especially RH down-type) in *specific* models: see, for example, reference [13] for one possible fit of c's to quark masses. In this paper, we will accept this mild tuning.

With this choice, the flavor-*preserving* shifts in Z couplings (see Eq. (6) without mixing angle) are marginal, i.e., $\sim$ a few 0.1%, for several TeV KK scale since $f\left(c_{L_\nu\, i}\right) \sim 1/\sqrt{\log\left(M_{Pl}/\mathrm{TeV}\right)}$ (see Eq. (2)). And 4-fermion operators induced by direct KK gauge exchange are quite safe for several TeV KK mass scale, using couplings in Eq. (5). However, with this size of f's, one problem is that the flavor-*violating* coupling to Z, $\delta g^Z_{\mu_L e_L}$ in Eq. (6), is still larger than in the case without neutrino masses (even with small mixing angle) – in the latter case, we get $f\left(c_{L_2}\right) \sim \sqrt{m_\mu/v} \sim 1/40$ for the case of similar RH and LH charged lepton profiles and $Y_5\sqrt{k} \sim O(1)$. Thus, including neutrino masses is still dangerous on a count similar to (II A) in section 3.3 even though count (I) is avoided. Thus we get the KK mass limit $> O(5)$ TeV from charged lepton flavor violation. Similarly, for the minimal case (0) considered earlier with the large LH charged lepton mixing angle, the KK mass limit will be even larger than that mentioned before, i.e., $> O(10)$ TeV if we insist on no tuning, i.e., choose $c_L \lesssim 1/2$.

Note that we also need to generate non-hierarchical *mass splittings* for neutrinos (in addition to mixing angles). We can achieve this goal by choosing $c_N \lesssim 1/2$, i.e., mild or no tuning of bulk masses for RH neutrinos giving non-hierarchical profiles near the TeV brane. Combined with the non-hierarchical LH neutrino profiles near the TeV brane (as above), the resulting Dirac neutrino masses will then be non-hierarchical, but too large since both RH and LH profiles generating neutrino masses near TeV brane are larger than those of charged leptons. However, we can include (a Planck/GUT-scale) Majorana mass term for RH neutrino on Planck brane and thereby use the see-saw mechanism [36, 16] to obtain very small neutrino masses (for other neutrino mass models

[20] As discussed below, we can invoke custodial symmetries to suppress shifts in couplings of fermions to Z in this case, but these can only protect *either* (not both) LH charged lepton or neutrino couplings to Z from being shifted.

[21] Explicitly, with dimensionless coefficient being $O\left(g_Z^2\right)$, such 4-fermion operators have to be suppressed by several TeV mass scale.

in warped extra dimension, see references [37, 20]).

5.1 Custodial protection

Next, we discuss how a custodial symmetry for the shift in the coupling of fermions to Z can relax the above tension for the choice of close-to-flat profiles for $l_\nu^{(0)}$ components of LH leptons. In particular, we

- choose $L_\nu : (\mathbf{2}, \mathbf{2})_\mathbf{0}$ which implies $T_{3L} = T_{3R}$ for this component of the LH charged leptons.

If we further choose the $5D$ $SU(2)_{L,\,R}$ couplings to be equal, then we realize the P_{LR} custodial symmetry [24] for this component of the LH charged leptons. Such a symmetry protects the couplings of SM Z to leptons (in the *weak* basis) from receiving a *non*-universal shift via zero-KK gauge mixing (again on account of the $l_\nu^{(0)}$ component) as follows – note that these shifts are flavor-*preserving*[22]. Recall that the couplings of LH charged leptons to KK Z and similarly to (KK) Z' induced by their $l_\nu^{(0)}$ components are *not* universal (although they have similar size). However, this symmetry enforces a cancellation between the non-universal contributions of KK Z and KK Z' in the mixing with $Z^{(0)}$ so that the *net* shift in the couplings of LH charged leptons (coming from their $l_\nu^{(0)}$ components) to SM Z is (approximately) universal, i.e., $\delta g^Z_{e_{L\,i} e_{L\,i}}$ is i-independent. Hence the resulting flavor-*violating* SM Z coupling (after rotating from weak to mass basis for leptons) is suppressed as well [23]. Note that we cannot simultaneously protect the (flavor-preserving) $Z\overline{\nu_L}\nu_L$ coupling from being shifted due to the $l_\nu^{(0)}$ component (due to $T_{3L} = +1/2 = -T_{3R}$ for ν_L). However, as mentioned above, in any case this effect is marginal (i.e., $\sim$ a few 0.1%) as long as $c_{L_\nu\,i} \sim 1/2$, i.e., $f\left(c_{L_\nu\,i}\right) \sim 1/\sqrt{\log\left(M_{Pl}/\mathrm{TeV}\right)}$ in Eq. (6) (without mixing angle).[24] Also, note that the $\delta g^Z_{\mu_L e_L}$ from $l_e^{(0)}$ component is *not* protected (since L_e must transform differently under $SU(2)_R$ than L_ν, i.e., it must have $T_{3R} \neq -1/2$, in order to decouple LH neutrino and charged lepton mixings), but anyway this effect is of similar size to the case without neutrino masses and safe since we can choose $f\left(c_{L_e\,2}\right) < 1/\sqrt{\log\left(M_{Pl}/\mathrm{TeV}\right)}$.

We would like to emphasize that this mechanism to suppress flavor-violating couplings to Z does *not* require the profiles at the TeV brane ($f\left(c_{L_\nu\,i}\right)$'s) and hence the couplings to KK Z (see Eq. (5)) to be universal *at all*[25], but rather relies up on *cancellations* between KK Z and KK Z'

[22] due to the couplings of leptons to *KK* Z, Z' being *diagonal* in this basis, as mentioned earlier.

[23] It is clear that even in presence of off-diagonal coupling of leptons (in the weak basis) to *KK* Z, Z' induced via brane-localized kinetic terms (or zero-KK fermion mixing due to Higgs vev), this custodial protection for flavor-violating lepton couplings to *SM* Z still works since it is the result of a cancellation between KK Z and Z'. Similarly, there is a cancellation between the contributions of various KK fermions to the shift in coupling to SM Z from zero-KK fermion mode mixing (see top left-hand side of Fig. 1) so that this effect also enjoys custodial protection.

[24] There is also a shift in charged current lepton couplings (vs. those for quarks) due to the mixing of KK and zero-mode W (especially due to $l_\nu^{(0)}$ component of SM lepton), but again this effect is marginal.

[25] again, we are assuming these f's are non-hierarchical in order to obtain large neutrino mixing, but still differing by $\sim O(1)$ factors.

contributions (each of which is *non*-universal) to the shifts in couplings of fermions to Z . In this sense this mechanism to suppress flavor-violating coupling to Z is quite distinct from the idea of $5D$ flavor symmetries which set c's (and thus f's) to be degenerate. Hence, with $5D$ flavor symmetries, couplings of leptons to KK Z (see Eq. (5)) and thus the contribution of KK Z to the shift in the coupling of fermions to SM Z is *by itself* universal, giving flavor-preserving Z couplings after going from weak to mass basis. Note that this result applies also to KK Z' contributions, i.e., it is valid *separately* for KK Z and KK Z', unlike for the custodial symmetry case considered here.

Direct KK Z, Z' exchange: a detailed analysis of this effect (including the effects of Z' which have not been calculated before[26]) is beyond the scope of this paper, but it suffices to note that this effect does not enjoy custodial protection (unlike the effect of mixing of KK Z with zero-mode Z). Moreover, due to the $l_\nu^{(0)}$ component of LH lepton with $c_{L_\nu} \sim 1/2$, i.e., $f\left(c_{L_\nu}\right) \sim 1/\sqrt{k\pi R}$, this effect can be enhanced (for the LH leptons only) compared to the case studied in reference [15] without neutrino masses. Specifically, with RH and LH charged lepton profiles being similar, we get $f^2\left(c_{L\,2}\right) \sim m_\mu/v \sim 1/1700$ for $Y_5\sqrt{k} \sim O(1)$ in the latter model. So, ratio of direct KK Z exchange in the two models is $\sim \sin^2\alpha \times 1700/(k\pi R)$ based on Eq. (9), where $\sin\alpha$ is the admixture of $l_\nu^{(0)}$ in the SM LH lepton as in Eq. (13).[27] As mentioned in section 3.1.1, direct KK Z exchange is suppressed compared to Z exchange in the model without neutrino masses by $\sim k\pi R$ and latter is on the edge of data for $M_{KK} \sim O(5)$ TeV. Thus we see that direct KK Z exchange in the model under consideration here is *marginal* if $\sin\alpha \sim 1$ and $M_{KK} \sim O(5)$ TeV.

Choice of L_e representations (responsible for charged lepton masses): one possibility is L_e: $(\mathbf{2},\mathbf{1})_{\mathbf{-1/2}}$ and E: $(\mathbf{1},\mathbf{2})_{\mathbf{-1/2}}$ as in **case (2)** above. The KK mass can then be as small as $O(5)$ TeV in case (2), even without any large tuning of c_{L_ν} in order to obtain large LH neutrino mixing angles. If we allow tuning of c_{L_ν}'s, we already saw in the beginning of section 5 that KK mass limit is same as that without neutrino masses, i.e., $\sim O(5)$ TeV, as long as we decouple LH charged lepton and neutrino mixings. Recall that case (1) also has small LH charged lepton mixing angle so that the KK mass limit can also be $\sim O(5)$ TeV, but this case does not have the custodial protection. So in case (1), we need to choose $c_{L_\nu} > 1/2$ in order to suppress $\delta g^Z_{\mu_L e_L}$ from $l_\nu^{(0)}$ component, implying that we need tuning for obtaining large LH neutrino mixing angle. A stronger limit on KK scale results if we instead choose $c_{L_\nu} \sim 1/2$ (i.e., no tuning) in case (1).

[26]although the couplings of Z' to quarks, relevant for μ to e conversion in nuclei, are expected to be negligible.

[27]Note that, as mentioned earlier, we assume that the couplings of leptons (in weak basis) to *KK* Z, Z' are (approximately) diagonal (but non-universal) in generation space. So, the *off*-diagonal couplings to KK Z, Z' giving flavor violation arise only after rotating to mass basis: the charged lepton mixing angle entering this effect in the cases we are considering here is the same as in the case without neutrino masses (i.e., this angle is small).

5.1.1 Best case scenario

So far, we have been able to obtain a *similar* level of charged lepton flavor violation (and hence $\sim O(5)$ TeV KK scale) as in the case without neutrino masses discussed in reference [15]. In fact, we can obtain *more* safety (relative to the case without neutrino masses in reference [15]) using **case (3)** above which has $T_{3L} = T_{3R} = 0$ for RH charged leptons. Such a choice of quantum numbers results in custodial protection (P_C symmetry [24]) for non-universal shifts in couplings of Z to RH charged leptons (in weak basis). As for the P_{LR} custodial symmetry discussed before, there is is a cancellation between the non-universal contributions of KK Z and (KK) Z' in the mixing with zero-mode Z[28], but we do not need the $5D$ $SU(2)_{L,\,R}$ gauge couplings to be equal in this case (unlike for P_{LR} symmetry). The idea then is to

- increase *all* $f(c_E)$'s by, say, $2\sqrt{2}$ compared to the case in reference [15] – note that flavor-*violating* coupling to Z, i.e., $\delta g^Z_{\mu_R e_R}$ (after rotating from weak to mass basis), is also protected by this custodial symmetry and
- reduce *all* $f(c_{L_e\,i})$ by $\sim \sqrt{2}$ – recall that $\delta g^Z_{\mu_L e_L}$ resulting from L_e component is not protected since we have $T_{3L} \neq T_{3R}$ for this component of LH charged lepton.

Hence, we can reduce Y_5 by ~ 2, keeping charged lepton masses fixed. Then both the tree-level[29] and loop amplitudes are reduced by ~ 4 (see Eqs. (6) and (10)) compared to the case in reference [15]. KK mass scale can then be smaller by ~ 2, i.e., $O(2.5)$ TeV.[30] If we keep increasing $f(c_E)$'s even more, then, eventually, RH charged lepton flavor violation from *direct* KK Z exchange (which is *not* protected) becomes relevant. Thus, we conclude that $\sim O(3)$ TeV KK scale can be consistent in case (3) with both charged lepton flavor violation and neutrino masses and with at most mild tuning of c_{L_ν}'s in order to obtain large neutrino mixing.[31]

5.1.2 A case with custodial protection, but *no* decoupling of mixing angles

In order to illustrate the independence of the above two mechanisms, namely, decoupling of large neutrino mixing from charged lepton sector (discussed in section 4) and custodial symmetry studied in this section, we consider a final case with only one $5D$ multiplet for LH leptons, but choose

[28] again, a similar effect occurs for zero-KK mode fermion mixing.

[29] Again, the LH contribution of Eq. (6) is suppressed only by ~ 2, but there is no RH contribution due to custodial protection, giving another reduction by factor of 2.

[30] We chose hierarchies in LH charged lepton profiles (at the TeV brane) from $l_e^{(0)}$ component to be similar to those of RH charged leptons, i.e., both RH and LH charged lepton $1-2$ mixing angle $\sim \sqrt{m_e/m_\mu}$, as in reference [15]. It is easy to check that such a choice minimizes the constraint from $\mu \to e\gamma$.

[31] Based on the previous discussion, it can be seen that a *very* mild tuning of $\sin\alpha$ is required to make the effect of direct KK Z exchange, coming from $l_\nu^{(0)}$ component of LH leptons, marginal for $\sim O(3)$ TeV KK scale.

L : $(\mathbf{2},\mathbf{2})_\mathbf{0}$ for custodial protection for LH charged leptons. With E : $(\mathbf{1},\mathbf{3})_\mathbf{0}$ and N : $(\mathbf{1},\mathbf{1})_\mathbf{0}$, we get

$$\text{Case (4)}: \quad Y_E : \overline{(\mathbf{2},\mathbf{2})_\mathbf{0}}(\mathbf{1},\mathbf{3})_\mathbf{0}(\mathbf{2},\mathbf{2})_\mathbf{0} \quad Y_N : \overline{(\mathbf{2},\mathbf{2})_\mathbf{0}}(\mathbf{1},\mathbf{1})_\mathbf{0}(\mathbf{2},\mathbf{2})_\mathbf{0} \tag{21}$$

The large, i.e., $O(1)$, mixing both for LH charged leptons and neutrinos due to the choice of one $5D$ L multiplet implies that we must choose Y_5 smaller (i.e., $\sim O(1/4)$) so that $\sim O(5)$ TeV KK mass scale can be allowed by $\mu \to e\gamma$ (see Eq. (10)). In order to compensate the effect of smaller Y_5 in τ Yukawa coupling, we can then increase $f\,(c_{E\,3})$ such that we are on the edge of the $\delta g^Z_{\tau_R\tau_R}$ constraint for several TeV KK mass scale, i.e., $f\,(c_{E\,3}) \sim 1/\sqrt{k\pi R}$ (recall that we do *not* have custodial protection for RH charged lepton couplings to Z in this case since $T_{3R} = -1$). Even with this extreme choice of $f\,(c_{E\,3})$, we still need $f\,(c_L) \sim 1/\sqrt{k\pi R}$, i.e., $c_L \sim 1/2$ in order to obtain m_τ. Anyway, $c_L \sim 1/2$ is favored by the desire to obtain large LH mixing angles with no tuning.[32] We can check that the tree-level $\delta g^Z_{\mu e}$ is quite safe, due to custodial protection for LH contribution and due to (very) small mixing angle for RH contribution (even for such large $f\,(c_{E\,3})$).

Based on the previous discussion, it is clear that the flavor violation from direct KK Z exchange in this case violates the experimental constraint by $\sim O(10)$ due to mixing angle being larger than in cases (2) and (3) discussed above. A mild tuning $\sin^2\alpha \sim O(0.1)$ can make this effect marginal for $M_{KK} \sim O(5)$ TeV in case (4). Note that we could also have invoked $\sin\alpha \ll 1$ in order to obtain a suppression for Z exchange (instead of using custodial protection), but clearly we would have needed significant tuning in this case.

The various possibilities discussed in this paper are summarized in table 1. Clearly, other possibilities with low KK scale can be constructed from *combinations* of the cases presented in this table.

6 Conclusions and Outlook

As we eagerly await the start of the LHC, where new physics at the TeV scale related to Planck-weak hierarchy of the SM might be discovered, it is interesting to study whether clues of *flavor* hierarchy of the SM could lie in this physics. In this paper, we considered one such possibility, namely, the framework of a warped extra dimension with the SM gauge and fermion fields propagating in the bulk. The flavor hierarchy of the SM can be accounted for in this framework using profiles for the SM fermions in the bulk, but the flip side is the resulting flavor violation from KK modes. Even though there is an automatic GIM-type mechanism, the limit on KK mass scale from flavor

[32] Also, we cannot choose $c_L < 1/2$ in spite of custodial protection for LH charged lepton coupling to Z since we do not simultaneously have such protection for ν_L couplings.

Table 1: The representations $\mathbf{r_{L_e}}$ and $\mathbf{r_{L_\nu}}$ under the bulk gauge symmetry, $SU(2)_L \times SU(2)_R \times U(1)_X$, for the two components of LH leptons, $l_e^{(0)}$ and $l_\nu^{(0)}$, respectively, discussed in the text. The "Y" and "N" in 3rd column convey whether charged lepton mixing angle is small or not. Similarly they convey whether the case has custodial symmetry for the $l_\nu^{(0)}$ component of LH lepton with non-hierarchical profiles at the TeV brane (which give neutrino masses with large mixing) or not (4th column) and finally custodial symmetry for RH charged lepton multiplet (5th column). The last column shows lower limit on M_{KK} from charged lepton flavor violation. For the cases with small LH charged lepton mixing, we are considering flavor-violating contributions from $l_e^{(0)}$ component of LH lepton (which give charged lepton masses) and RH charged lepton multiplets only, i.e., contribution from the non-hierarchical $l_\nu^{(0)}$ profiles is assumed to be negligible in these cases. This assumption is justified due to either (i) the choice of these profiles peaked near Planck brane, which requires tuning of bulk masses in order to obtain large neutrino mixing or (ii) presence of custodial protection (i.e., "Y" in 4th column) for the case of close-to-flat profiles, which does not require tuning.

$\mathbf{r_{L_e}}$	$\mathbf{r_{L_\nu}}$	small mixing angle?	L_ν custodial?	RH custodial?	lower limit on M_{KK}
$(\mathbf{2},\mathbf{1})_{\mathbf{-1/2}}$	$(\mathbf{2},\mathbf{1})_{\mathbf{-1/2}}$	N	N	N	$\sim O(10)$ TeV
$(\mathbf{2},\mathbf{1})_{\mathbf{-1/2}}$	$(\mathbf{2},\mathbf{5})_{\mathbf{-1/2}}$	Y	N	N	$\sim O(5)$ TeV
$(\mathbf{2},\mathbf{1})_{\mathbf{-1/2}}$	$(\mathbf{2},\mathbf{2})_{\mathbf{0}}$	Y	Y	N	$\sim O(5)$ TeV
$(\mathbf{2},\mathbf{2})_{\mathbf{-1}}$	$(\mathbf{2},\mathbf{2})_{\mathbf{0}}$	Y	Y	Y	$\sim O(3)$ TeV
$(\mathbf{2},\mathbf{2})_{\mathbf{0}}$	$(\mathbf{2},\mathbf{2})_{\mathbf{0}}$	N	Y	N	$\sim O(5)$ TeV

violation in both the quark and charged lepton sector (with*out* considerations of neutrino data) is still $\sim O(5)$ TeV.

Moreover, if we include the neutrino data, then the charged lepton flavor violation tends to be enhanced by the large charged current mixing required to account for neutrino oscillations. The point is that, in the minimal model, the mixings are similar for LH charged leptons and neutrinos, being dictated by LH profiles (at the TeV brane) which are same for the two sectors. Hence, the limit on gauge KK mass scale from charged lepton flavor violation *when combined with neutrino data* is *larger than* $\sim O(5)$ TeV, making any signals at the LHC from direct production of gauge KK modes unlikely.

In this paper, we presented new mechanisms which can suppress charged lepton flavor violation in this framework. The central point is to use less minimal representations for leptons under the extended $5D$ gauge symmetry[33], allowing mixing angles to be (simultaneously) small and large in the LH charged lepton and neutrino sectors, respectively. The trick is that the LH lepton zero-mode is actually a combination of two zero-modes with different profiles, one giving charged lepton masses and the other neutrino masses. Furthermore, such representations can lead to custodial

[33]such an extension of $5D$ gauge symmetry is motivated for satisfying EWPT.

protection for the shift in couplings of charged leptons to Z (ala $Zb\bar{b}$) and hence suppress charged lepton flavor violation from tree-level exchange of Z.

The bottom line is that $\sim O(3)$ TeV gauge KK mass scale might then allowed by charged lepton flavor violation, including neutrino masses and without any particular structure in the $5D$ flavor parameters. However, charged lepton flavor violation is still not "super-safe" (unlike in some models with $5D$ flavor symmetries) so that the upcoming lepton flavor violation experiments (MEG at PSI [38], PRIME at JPARC [39] and the proposed mu2e experiment at Fermilab [40]) should see a signal. The situation is similar to reference [15] without considerations of neutrino masses since the two issues of charged lepton flavor violation and neutrino masses are now decoupled. Also, with $\sim O(3)$ TeV gauge KK scale, signals form direct production of these KK modes at the LHC are then still viable [41].

6.1 Other applications

We would like to emphasize that the mechanism discussed in this paper is quite general as we discuss below with several examples.

Quark sector in warped extra dimensional framework: the mechanism for decoupling of mixing angles of the LH charged leptons and neutrinos can be applied to quark $SU(2)_L$ doublets as well for the warped extra-dimensional scenario. In particular, we can arrange for LH down-type quark mixing to be *parametrically* smaller than LH up-type quark mixing – the latter would then have to entirely account for the CKM mixing. Thus flavor-violating effects involving LH down-type quarks can be suppressed compared to the minimal models, where the LH down-type and up-type quark mixings are similar (just like for LH charged leptons and neutrinos) and hence of CKM-size.

However, the dominant constraint on gauge KK scale from flavor violation in the quark sector comes from contributions to ϵ_K involving *both* LH and RH down-type quarks. While LH down-type quark mixings can be suppressed using the trick used for leptons here, it is easy to see that the the RH down-type quark mixings are enhanced compared to minimal models[34] so that this mixed contribution to ϵ_K is not affected. However, the dominant contribution to $B_{d,\,s}$ mixing does come from operators with LH down-type quarks only and hence it can be suppressed using the mechanism of decoupling LH up and down-type quark mixing angles. Of course, the constraint on KK mass scale from these systems is (generically) weaker than the one from ϵ_K. In short, it seems difficult to *fully* ameliorate the constraints from quark sector flavor violation using this mechanism.

Combining with other proposals within warped extra dimensional framework: we have presented the new mechanisms for suppressing charged lepton flavor violation in the warped

[34]just like we found for charged leptons in section 3.3 that enhancement of LH mixing angles implies reduction in RH mixing angles.

extra dimensional framework in a manner showing their independence from other ideas in the literature. However, it is clear that these mechanisms can actually be combined with other ideas. For example, reference [22] proposed obtaining (naturally) small and large mixing angles for LH charged leptons and neutrinos, respectively, even with *minimal* choice of representations of the bulk gauge symmetry, in a framework where neutrinos are Dirac particles and with a bulk Higgs. In this framework, we choose non-degenerate $c_L > 1/2$, i.e., with LH lepton profiles being hierarchical near the TeV brane and non-hierarchical near the Planck brane. The point is that we can then obtain small charged lepton mixing angles (as usual) since charged lepton masses are dominated by overlap with Higgs near the TeV brane, whereas neutrino masses can be dominated by overlap of profiles near the Planck brane thus giving large neutrino mixings. Thus, even without using the new $SU(2)_R$ representations (i.e., the two mechanisms of this paper), the lepton flavor violation constraints in this framework for neutrino masses reduce to the case without neutrino masses studied in [15], i.e., the gauge KK mass limit is $\sim O(5)$ TeV. In addition, the mild tuning of c_L's required in the models considered here in order to obtain large neutrino mixings is avoided in the idea of reference [22] .

Interestingly, we can add the custodial protection for charged lepton couplings to Z to the above idea. Namely, we move either LH or RH charged lepton profile closer to the TeV brane (relative to the choice of same RH and LH profiles), invoking custodial symmetry to protect tree-level flavor violation via Z exchange from this chirality. Simultaneously, we move the profile of the other chirality (which does not enjoy custodial protection) away from the TeV brane (thus reducing tree-level μ to e conversions) in such a way as to allow a reduction in $5D$ Yukawa coupling and thus suppressing, in turn, loop-induced $\mu \to e\gamma$ as well. We note that

- such a strategy (along the lines discussed in section 5.1.1) can allow us to lower the limit (from lepton flavor violation) on the gauge KK scale in this framework from $\sim O(5)$ TeV (which is the value without custodial protection) down to $\lesssim O(3)$ TeV.

Similarly, these mechanisms can be suitably combined with $5D$ flavor symmetries. We will leave these directions for future work.

Beyond applications to the warped extra dimensional framework: these mechanisms might enable suppression of flavor violation (especially in charged lepton sector) in other extra-dimensional models which explain flavor hierarchy via profiles, as long as there is an extended gauge symmetry to play the decoupling trick. In particular, another framework where the origins of flavor leave their imprint on physics at the TeV scale (i.e., within LHC reach) is the recently proposed $5D$ flavorful SUSY [42]. This $5D$ set-up can be quite similar to that considered in this paper, namely Higgs localized on one brane in an extra dimension with light fermion profiles being peaked near

the other end of the extra dimension – the smallness of the fermion profiles near the Higgs brane then account for the lightness of these fermions.

More importantly, SUSY breaking can occur on the Higgs brane in this framework such that the non-universalities/mixing among squarks and sleptons are governed by the (s)fermion profiles near Higgs brane. Thus the structure of squark and slepton masses is correlated with the SM Yukawa couplings, possibly suppressing SUSY contributions to flavor violation at least for the 1st/2nd generation. This effect for $5D$ flavorful SUSY is the analog of the GIM-like mechanism for KK contributions considered here – of course, the *actual* KK contributions could be much smaller for $5D$ flavorful SUSY due to higher compactification scale with the resulting hierarchy between that scale and the weak scale being explained by SUSY. However, charged lepton flavor violation in $5D$ flavorful SUSY could be enhanced due to large neutrino mixing just like discussed here. It will be interesting to further study the mechanisms for suppressing flavor violation discussed in this paper in the context of $5D$ flavorful SUSY.

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